

MATHEMATICS

MAXIMUM MARKS: 80

TIME ALLOWED: THREE HOURS

(CANDIDATES ARE ALLOWED ADDITIONAL 15 MINUTES FOR ONLY READING THE PAPER. THEY MUST NOT START WRITING DURING THIS TIME.)

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- *This Question Paper consists of three sections A, B and C.*
 - *Candidates are required to attempt all questions from Section A and all questions EITHER from Section B OR Section C.*
 - *Section A: Internal choice has been provided in two questions of two marks each, two questions of four marks each and two questions of six marks each.*
 - *Section B: Internal choice has been provided in one question of two marks and*
 - *one question of four marks.*
 - *Section C: Internal choice has been provided in one question of two marks and*
 - *one question of four marks.*

- All working, including rough work, should be done on the same sheet as, and adjacent to
- the rest of the answer.
- The intended marks for questions or parts of questions are given in brackets [].
- Mathematical tables and graph papers are provided.

SECTION A - 65 Marks

Question 1

In subparts (i) to (xi) choose the correct options and in subparts (xii) to (xv), answer the questions as instructed.

(i) If A and B are square matrices of order 2 such that $AB = A$ and $BA = B$, then $A^2 + B^2$ is equal to:

- (a) $A + B$
- (b) $2AB$
- (c) $2BA$
- (d) $A - B$

(ii) The degree of the differential equation $\frac{d^2y}{dx^2} + \sin\left(\frac{dy}{dx} + y\right) = 0$ is:

- (a) 2
- (b) 1
- (c) Not defined
- (d) 0

(iii) The derivative of $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$ with respect to $\tan^{-1}x$ is:

- (a) 0

(b) 1

(c) 2

(d) $\frac{1}{2}$

(iv) Assertion: The determinant of a skew-symmetric matrix of odd order is always 0.

Reason: For any square matrix A , $|A| = |A'|$. (Analysis)

(a) Both Assertion and Reason are true, and Reason is the correct explanation for Assertion.

(b) Both Assertion and Reason are true, but Reason is not the correct explanation for Assertion.

(c) Assertion is true and Reason is false.

(d) Assertion is false and Reason is true.

(v) Three distinct numbers are chosen randomly from the set $\{1, 2, 3, \dots, 10\}$. The probability that they are in Arithmetic Progression is:

(a) $\frac{1}{10}$

(b) $\frac{2}{15}$

(c) $\frac{1}{12}$

(d) $\frac{1}{15}$

(vi) If $A = \begin{bmatrix} 0 & 2 \\ 3 & -1 \end{bmatrix}$, then the value of

$A^2 - 3A - 10I$ is: (Understanding)

(a) $\begin{bmatrix} -4 & -8 \\ -12 & 0 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}$

(d) $\begin{bmatrix} -10 & 0 \\ 0 & -10 \end{bmatrix}$

(vii) Statement 1: The function $f(x) = |x - 2| + |x - 3|$ is continuous for all $x \in \mathbb{R}$.

Statement 2: The sum of two continuous functions is continuous.

(a) Statement 1 is true and Statement 2 is false.

(b) Statement 2 is true and Statement 1 is false.

(c) Both the statements are true.

(d) Both the statements are false.

(viii) If $f(x) = \log(\log x)$, then $f'(e)$ is equal to:

(a) e

(b) $\frac{1}{e}$

(c) 1

(d) 0

(ix) Statement 1: A relation R on set A is an equivalence relation if it is reflexive and symmetric.

Statement 2: For a relation to be an equivalence relation, it must also be transitive.

(a) Statement 1 is true and Statement 2 is false.

(b) Statement 2 is true and Statement 1 is false.

(c) Both the statements are true.

(d) Both the statements are false.

(x) Assertion: $\sin^{-1}\left(\frac{3}{5}\right) + \cos^{-1}\left(\frac{12}{13}\right) = \tan^{-1}\left(\frac{63}{16}\right)$

Reason: $\tan^{-1} x + \tan^{-1} y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)$ for $xy < 1$.

(a) Both Assertion and Reason are true and Reason is the correct

explanation for Assertion.

(b) Both Assertion and Reason are true but Reason is not the correct explanation for Assertion.

(c) Assertion is true and Reason is false.

(d) Assertion is false and Reason is true.

(xi) If $P(A) = 0.6$, $P(B) = 0.5$ and $P(A \cap B) = 0.4$, then $P(A' | B')$ is:

(a) $\frac{1}{3}$

(b) $\frac{1}{5}$

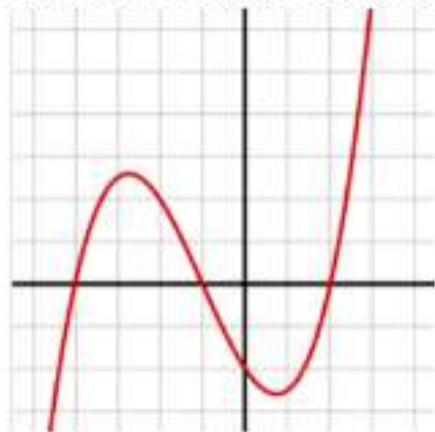
(c) $\frac{2}{5}$

(d) $\frac{3}{5}$

(xii) If the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$, find the value of $|\text{Adj}(\text{Adj} A)|$

(xiii) Check whether the relation $R = \{(1,1), (2,2), (3,3), (1,2), (2,1)\}$ on the set $A = \{1,2,3\}$ is an equivalence relation or not. Justify your answer.

(xiv) The graph of a polynomial function $y = p(x)$ is shown below. Is the function one-one? Justify your answer.



(xv) A box contains 10 bulbs, 4 of which are defective. If 3 bulbs are drawn at random without replacement, what is the probability that at least one is defective?

Question 2

[2]

(i) If $x^y = y^x$, then prove that $\frac{dy}{dx} = \frac{y(x \log y - y)}{x(y \log x - x)}$.

OR

(ii) Find the interval in which the function $f(x) = x^3 - 6x^2 + 9x + 15$ is strictly increasing.

Question 3

[2]

Find the points on the curve $x^2 + y^2 - 2x - 3 = 0$ at which the tangents are parallel to the x-axis.

Question 4

[2]

Show that the solution of the differential equation $\frac{dy}{dx} = \frac{y}{x} + \tan\left(\frac{y}{x}\right)$ is $\sin\left(\frac{y}{x}\right) = Cx$.

Question 5

[2]

(i) If $\int e^x \left(\frac{1 - \sin x}{1 - \cos x} \right) dx = -e^x \cot\left(\frac{x}{2}\right) + C$, find the value of the constant of integration when $x = \frac{\pi}{2}$. (Evaluate)

OR

(ii) Evaluate: $\int_0^2 |x^2 + 2x - 3| dx$.

Question 6

[2]

Find the domain of the function f given by

$$f(x) = \frac{1}{\sqrt{[x]^2 - [x] - 6}}$$

Question 7 [4]

Solve for x : $\tan^{-1}(x+1) + \tan^{-1}(x-1) = \tan^{-1}\left(\frac{8}{31}\right)$.

Question 8 [4]

If $y = e^{a \cos^{-1} x}$, prove that $(1-x^2) \frac{d^2y}{dx^2} - x \frac{dy}{dx} - a^2 y = 0$.

Question 9 [4]

(i) Solve the differential equation: $(x^2 - y^2)dx + 2xydy = 0$, given that $y = 1$ when $x = 1$.

OR

(ii) Solve the differential equation: $\frac{dy}{dx} + y \sec x = \tan x$.

Question 10 [4]

(i) Three friends play a game where each simultaneously shows either a red or a blue card. The game rules are:

- If all three show the same color, nobody wins.
- If one person shows a different color from the other two, that person wins.

(Analysis)

(a) Find the probability that there is a winner in a round.

(b) Find the probability that it takes exactly 3 rounds to get a winner for the first time.

OR

(ii) A factory has three machines, M_1 , M_2 , and M_3 , producing 50%, 30%, and 20% of the total output respectively. Their

defective rates are 2%, 3%, and 5% respectively. An item is drawn at random and found to be defective.

- Represent the situation using probability tree diagram or otherwise.
- Find the probability that it was produced by machine M_2 .

Question 11

[6]

A company manufactures three types of products: P, Q, and R. The cost of raw materials, labor, and packaging for each unit is given below:

Product	Raw Material (₹)	Labor (₹)	Packaging (₹)
P	20	30	10
Q	30	20	20
R	40	40	30

The total cost incurred for manufacturing 10 units of P, 15 units of Q, and 20 units of R is ₹2050. For 20 units of P, 10 units of Q, and 15 units of R, it is ₹2150. For 15 units of P, 20 units of Q, and 10 units of R, it is ₹1900.

- Represent the problem as a system of linear equations.
- Find the cost of raw materials, labor, and packaging per unit using the matrix method.
- If the selling price of P, Q, R is ₹100, ₹110, and ₹150 respectively, which product gives the maximum profit per unit?

Question 12

[6]

(i) Evaluate: $\int \frac{x^2+1}{(x^2+4)(x^2+9)} dx$.

OR

(ii) Evaluate: $\int_0^{\pi} \frac{x \sin x}{1+\cos^2 x} dx$.

Question 13

[6]

(i) A right-circular cone is inscribed in a sphere of fixed radius R.

(a) Express the volume V of the cone in terms of its height h and R.

(b) Show that the volume is maximum when the height of the cone is $\frac{4R}{3}$.

(c) Find the maximum volume.

OR

(ii) The path of a projectile is given by the equation $y = x \tan \theta - \frac{gx^2}{2u^2 \cos^2 \theta}$, where u is the initial velocity and g is acceleration due to gravity.

(a) Find the value of $\frac{dy}{dx}$ at $x = 0$.

(b) Find the value of x for which the height y is maximum.

(c) Prove that the maximum height attained is $\frac{u^2 \sin^2 \theta}{2g}$.

Question 14

[6]

The number of emails received by a customer service department in an hour is a random variable X with the following probability distribution:

$X = x$	0	1	2	3	4
$P(X=x)$	0.1	k	0.2	$2k$	0.1

- Find the value of k .
- Construct the probability distribution table.
- Find the mean number of emails received per hour.
- If the department can handle a maximum of 3 emails per hour without delay, what is the probability that an hour will see delayed responses?

SECTION B - 15 Marks

Question 15

[5]

In subparts (i) and (ii) choose the correct options and in subparts (iii) to (v), answer the questions as instructed.

(i) Statement 1: The vectors $\vec{a} = 2\hat{i} + 3\hat{j} - \hat{k}$ and $\vec{b} = 4\hat{i} + 6\hat{j} + \lambda\hat{k}$ are parallel if $\lambda = -2$.

Statement 2: Two vectors are parallel if their cross product is the zero vector.

- Statement 1 is true and Statement 2 is false.
- Statement 2 is true and Statement 1 is false.
- Both the statements are true.
- Both the statements are false.

(ii) The equation of the plane passing through the point $(1, 2, 3)$ and perpendicular to the planes $2x - y + z = 5$ and $x + y + 2z = 7$ is:

- $x - 3y + z + 2 = 0$

(b) $3x - y - z - 2 = 0$

(c) $x - 3y + z - 2 = 0$

(d) $3x - y - z + 2 = 0$

(iii) If a line makes angles α, β, γ with the coordinate axes, prove that $\sin^2 \alpha + \sin^2 \beta + \sin^2 \gamma = 2$. [1]

(iv) Find the value of λ for which the points with position vectors $\hat{i} - \hat{j} + 3\hat{k}$, $2\hat{i} + \hat{j} + 2\hat{k}$, and $3\hat{i} + \lambda\hat{j} + \hat{k}$ are collinear. [1]

(v) Find the angle between the lines whose direction ratios are (1, 1, 2) and $(\sqrt{3} - 1, -\sqrt{3} - 1, 4)$. [1]

Question 16 [4]

(i) Find the shortest distance between the lines:

$$\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 4\hat{k}) \text{ and}$$

$$\vec{r} = (2\hat{i} + 4\hat{j} + 5\hat{k}) + \mu(3\hat{i} + 4\hat{j} + 5\hat{k}).$$

OR

(ii) Find the equation of the line passing through the point (1, 2, -4) and perpendicular to the two lines:

$$\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7} \text{ and } \frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$$

Question 17 [4]

(i) If $\vec{a}, \vec{b}, \vec{c}$ are unit vectors such that $\vec{a} + \vec{b} + \vec{c} = \vec{0}$, find the value of $\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$.

OR

(ii) For any three vectors $\vec{a}, \vec{b}, \vec{c}$, prove that $[\vec{a} + \vec{b}, \vec{b} + \vec{c}, \vec{c} + \vec{a}] = 2[\vec{a}, \vec{b}, \vec{c}]$.

Question 18

[4]

Using integration, find the area of the region bounded by the curve $y = x^3$, the line $y = 8$, and the y-axis.

SECTION C - 15 Marks

Question 19

[5]

In subparts (i) and (ii) choose the correct options and in subparts (iii) to (v), answer the questions as instructed.

(i) If the regression coefficients are 0.8 and 0.2, then the coefficient of correlation is:

- (a) 0.16
- (b) 0.40
- (c) 0.04
- (d) 0.32

(ii) The total cost function is given by $C = \frac{x^3}{3} - 3x^2 + 12x + 10$. The level of output where marginal cost is minimum is:

- (a) 1
- (b) 2
- (c) 3
- (d) 4

(iii) For 5 observations on x and y, $\sum x = 15$, $\sum y = 25$, $\sum xy = 90$, $\sum x^2 = 55$, $\sum y^2 = 135$. Find the regression coefficient b_{yx} . [1]

(iv) The total revenue from the sale of x units is given by $R(x) = 20x - \frac{x^2}{2}$. Find the marginal revenue when 5 units are sold. [1]

(v) The cost function for x articles is given by $C(x) = x^2 + 78x + 2000$. If they are sold at ₹200 per article, find the number of articles to be produced and sold for break-even. [1]

Question 20

[4]

(i) The two lines of regression are $3x + 2y - 26 = 0$ and $6x + y - 31 = 0$. Find the mean values of x and y and the variance of y given that the variance of x is 25.

OR

(ii) From the data given below, find the line of regression of y on x and hence estimate y when $x = 12$.

x	4	6	8	10
y	7	9	11	13

Question 21

[4]

(i) The demand function for a commodity is $p = 20 - 2x - x^2$. Find:

- The revenue function.
- The marginal revenue function.
- The output (x) where marginal revenue is zero.

OR

(ii) The Average Cost (AC) function for a firm is $AC = 2x + 5 + \frac{36}{x}$.

(a) Find the Total Cost (C) function and the Marginal Cost (MC) function.

(b) Show that $\frac{d}{dx}(AC) = \frac{MC - AC}{x}$.

Question 22

[4]

(i) A farmer has a field of 100 hectares. He decides to plant wheat and barley. The profit per hectare for wheat is ₹5000 and for barley is ₹3000. Wheat requires 3 units of fertilizer per hectare and barley requires 2 units. The total fertilizer available is 240 units. Formulate this as a Linear Programming Problem to maximize profit and solve it graphically.

OR

(ii) The feasible region for an LPP is given by the inequalities $x + y \leq 4$, $x \geq 0$, $y \geq 0$. The objective function is $Z = 3x + 4y$.

(a) Find the corner points of the feasible region.

(b) Find the maximum value of Z .

